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ECE 3306-D01
Submitted 12 June 2023

Project One: Three Phase Circuit

Part One: The Circuit

Initial Description:

The circuit's layout is a Y-to-Y with:

- Three 120 V sources @ 60 Hz w/ 120° phase difference between each phase (positive phase sequence)
- On each line:
  - A source impedance of 0.1 + j0.4 Ω
  - A line impedance of 2.9 + j1.6 Ω
  - A load impedance of 77 + j58 Ω

Calculating values for the circuit:

The real portion of each impedance will be characterized by a resistor with the given value. That is to say, the real portion of the source impedance will be characterized by a 0.1 Ω resistor, of the line impedance by a 2.9 Ω resistor, and of the load impedance by a 77 Ω resistor.

For the imaginary portion, an inductor will be used. To find the value of the inductor, we use the following equation:

$$Z = jL\omega \quad (1)$$

However, since we want to find the inductors' inductances in Henrys, we must rearrange equation 1 as such:

$$L = \frac{Z}{\omega} \text{ (2)}$$

The value of ω is constant for this problem:

$$\omega = 2\pi * 60 = 376.9911 \text{ (3)}$$

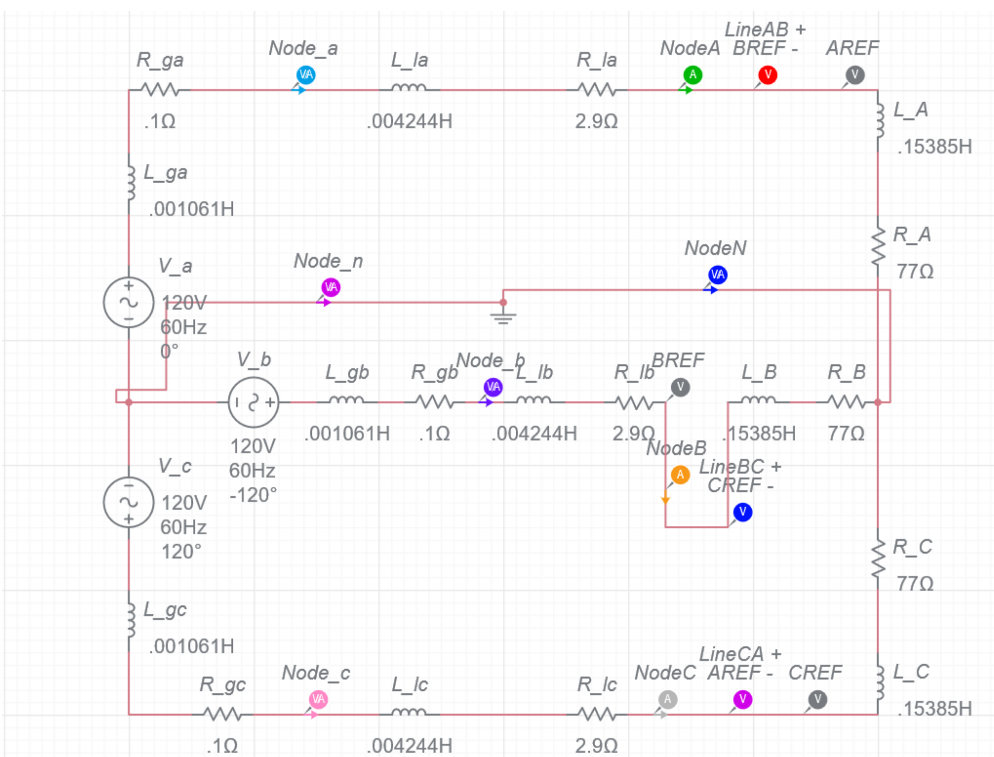
Thus, the value of each inductor, using equations 2 and 3, is as follows:

$$L_{source} = \frac{0.4}{\omega} = 0.00106\ H$$

$$L_{line} = \frac{1.6}{\omega} = 0.004244\ H$$

$$L_{load} = \frac{58}{\omega} = 0.15385\ H$$

At last, our circuit will end up looking like this:



To find the voltage drop across each impedance, we must first find the total impedance followed by the current. To find the total impedance, we add each component in series and then find the RMS of the real with the imaginary portion (we will begin with just the A phase):

$$Z = \sqrt{R^2 + X_L^2} \ (4)$$

$$Z_{TA} = \sqrt{80^2 + 60^2} = 100\ \Omega$$

Then, we will need to eventually know the lag caused by the impedance on the current compared to the voltage:

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) (5)$$

$$\phi_A = \tan^{-1}\left(\frac{60}{80}\right) = 36.87°$$

Now, we can calculate the current running through the A phase using simple Ohm's law:

$$I = \frac{V}{Z} \ (6)$$

$$\boldsymbol{I}_{TA} = \frac{\boldsymbol{V}_{TA}}{Z_{TA}} = \frac{120\angle 0°}{100\angle 36.87°} = 1.2\angle -36.87°\ A$$

Since we are here, we can quickly calculate the rest of the phases' currents using the above equation with the small modification of offsetting the voltage by +120° for the C phase and -120° for the B phase:

$$\boldsymbol{I}_{TB} = \frac{\boldsymbol{V}_{TB}}{Z_{TB}} = \frac{120\angle -120°}{100\angle 36.87°} = 1.2\angle -156.87°\ A$$

$$\boldsymbol{I}_{TC} = \frac{\boldsymbol{V}_{TC}}{Z_{TC}} = \frac{120\angle 120°}{100\angle 36.87°} = 1.2\angle 83.13°\ A$$

Now, with each of the phase currents calculated, we can do simple Ohm's law to find the voltage drop across each impedance. For brevity, I will merely mention that I found each impedance in Ohms using equation 4 and the lag caused by each impedance using equation 5.

$$\boldsymbol{V}_{gA} = \boldsymbol{I}_{TA}Z_{gA} = 1.2\angle - 36.87° * .4123\angle 75.96° = .49476\angle 39.21°\, V$$

$$\boldsymbol{V}_{lA} = \boldsymbol{I}_{TA}Z_{lA} = 1.2\angle - 36.87° * 3.312\angle 28.89° = 3.974\angle - 7.86°\, V$$

$$\boldsymbol{V}_{A} = \boldsymbol{I}_{TA}Z_{A} = 1.2\angle - 36.87° * 96.4\angle 36.99° = 115.68\angle 0.24°\, V$$

The same process is then used for the other two phases, using $\boldsymbol{I}_{TB}$ and $\boldsymbol{I}_{TC}$ instead of $\boldsymbol{I}_{TA}$. With those changes, the results are:

$$\boldsymbol{V}_{gB} = .49476 \angle - 80.91°\, V$$

$$\boldsymbol{V}_{lB} = 3.974\angle - 127.98°\, V$$

$$\boldsymbol{V}_{B} = 115.68\angle - 119.88°\, V$$

$$\boldsymbol{V}_{gC} = .4947\angle 159.09°\, V$$

$$\boldsymbol{V}_{lC} = 3.974\angle 112.02°\, V$$

$$\boldsymbol{V}_{C} = 115.68\angle 120.12° V$$

Note that $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ are the phase voltages across the loads.

Now, to find the line voltages we apply the following equation to each of the phase voltages:

$$\boldsymbol{V}_{line} = \boldsymbol{V}_{phase}\sqrt{3}\angle 30° \; (7)$$

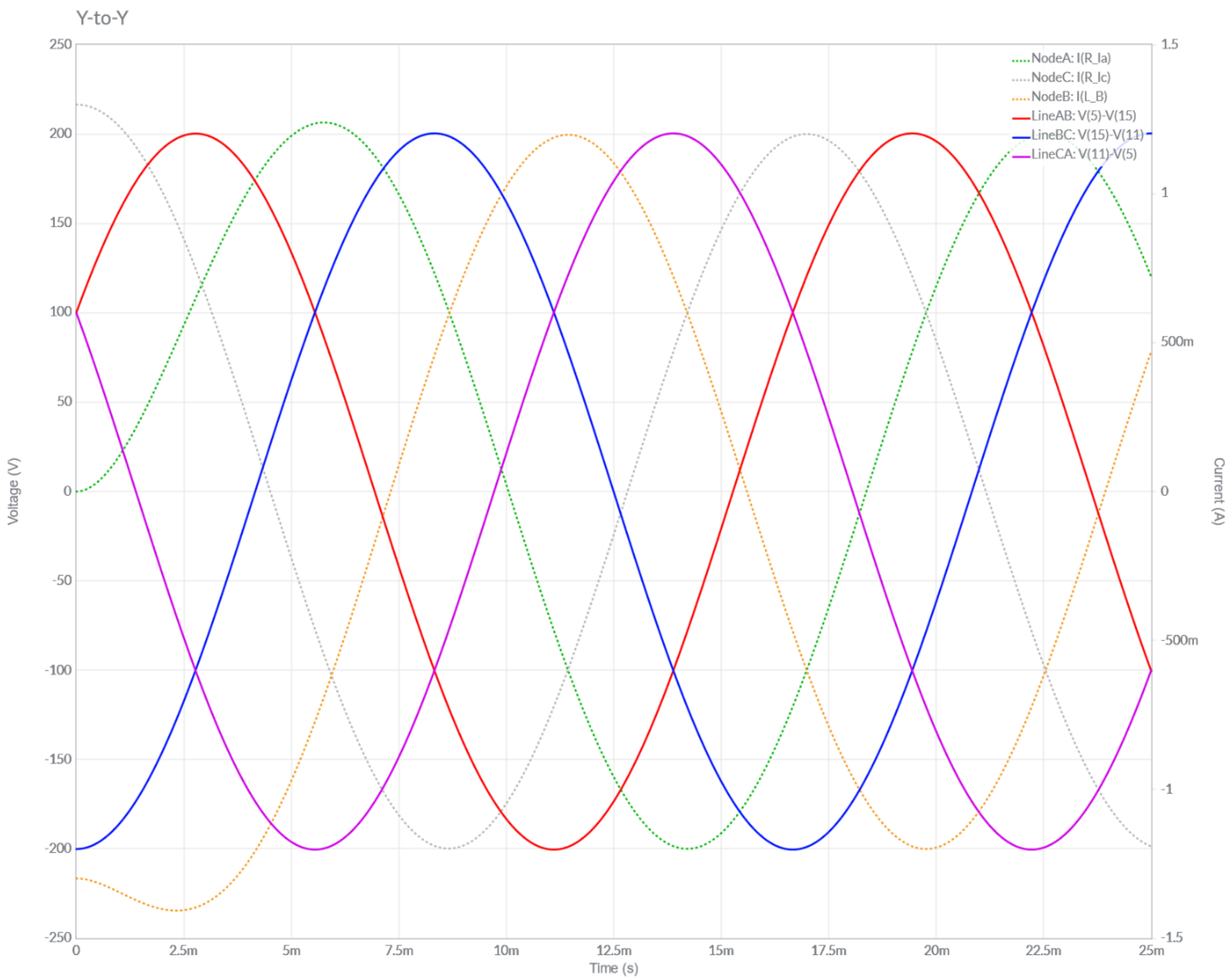
Thus, we get the following results:

$$\boldsymbol{V}_{AB} = 115.68\angle 0.24° * \sqrt{3}\angle 30° = 200.36\angle 30.24°\, V$$

$$\boldsymbol{V}_{BC} = 200.36\angle -89.88° \, V$$

$$\boldsymbol{V}_{CA} = 200.36\angle 150.12° \, V$$

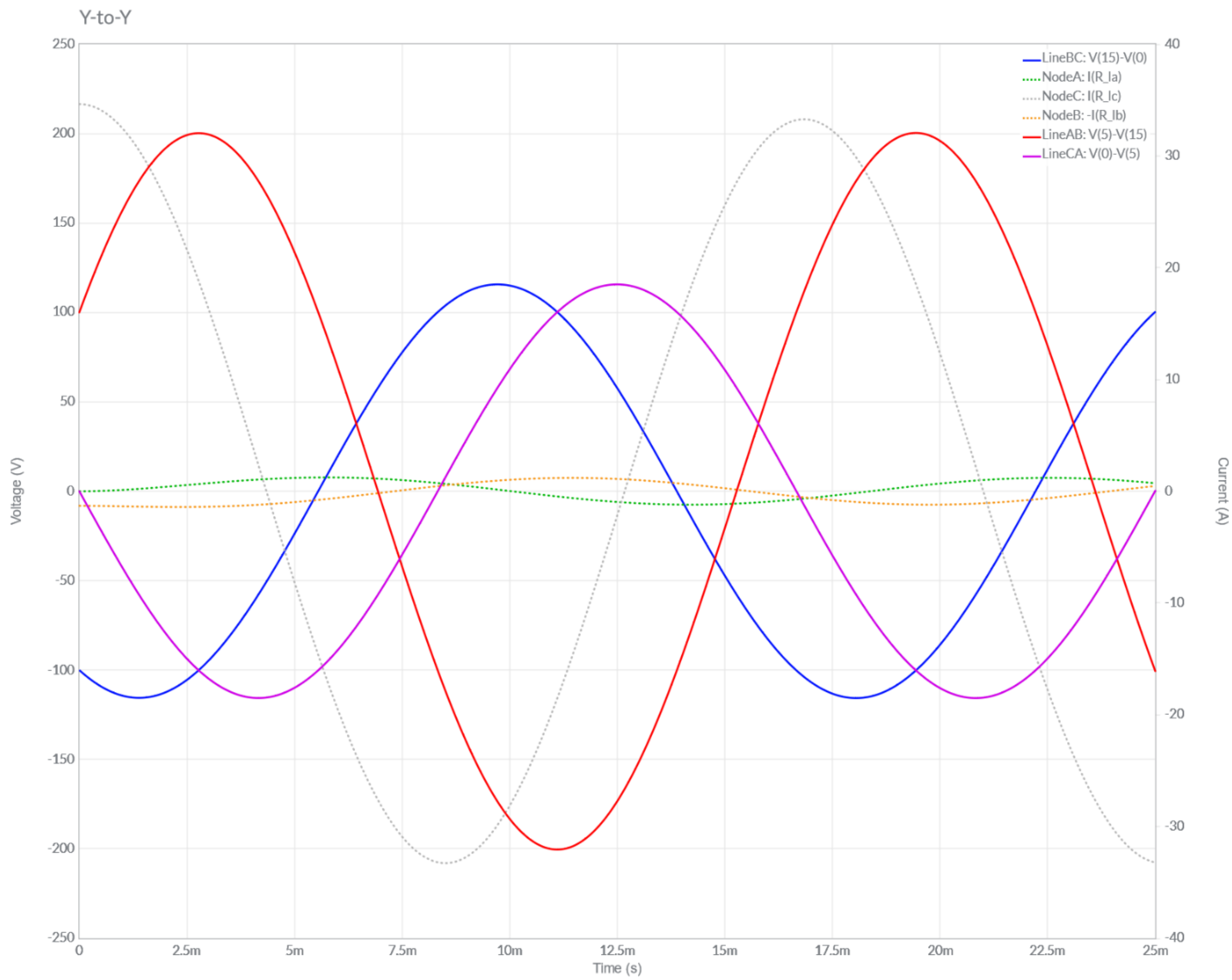
The signals of the circuit are as follows:



A note for all graphs in this report:
The voltage portion of the graphs signify the voltages measured at the load terminals.

Part Two: Introducing Faults- Short Circuit

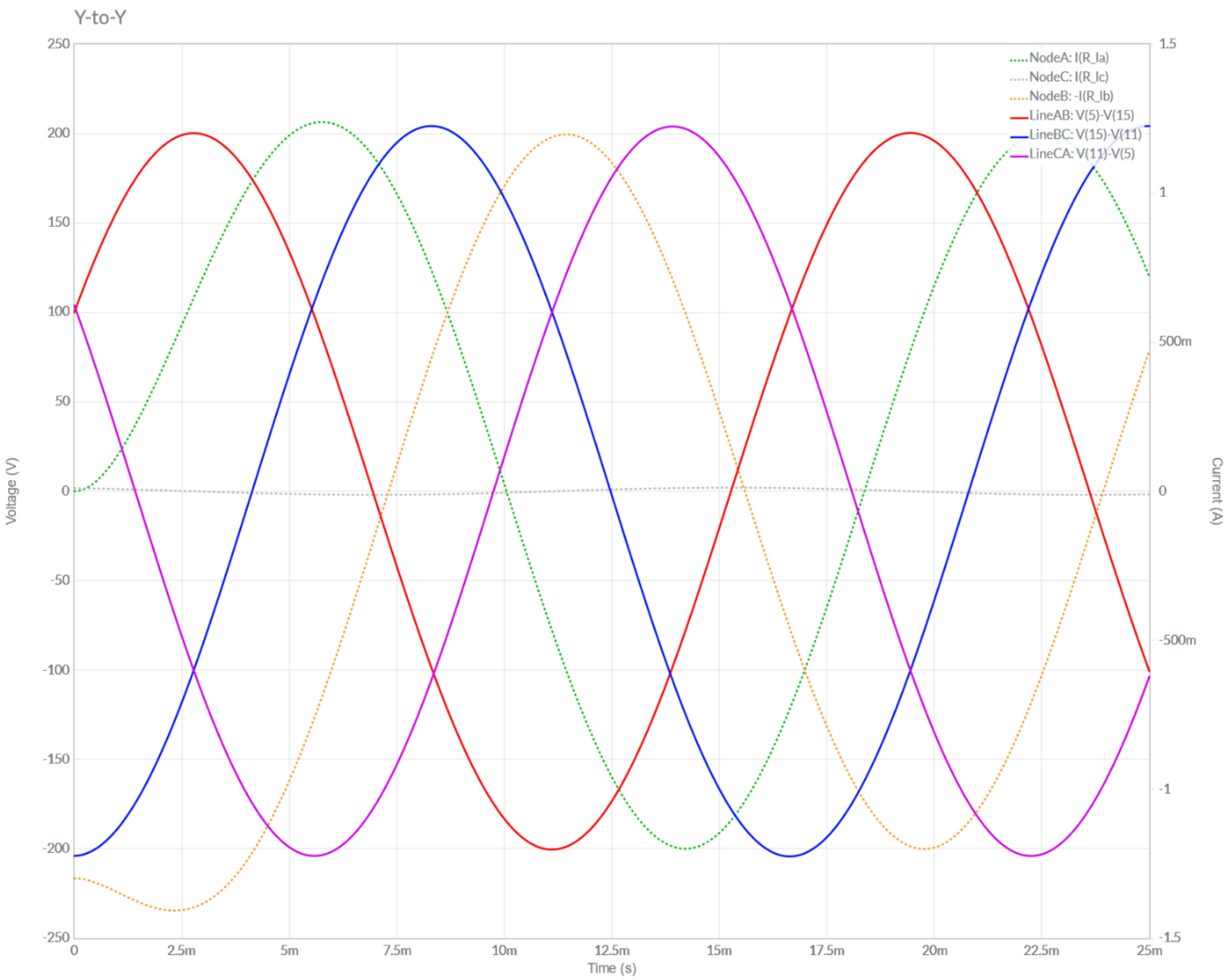
In this simulation, I opted to short phase C's load. The results are as follow:



In comparison to the original circuit, the main difference is the C-phase line current, soaring to a max/min value of ±33.28 A, yet the A and B phase line currents remain the same ±1.2 A. Voltage is also altered for the BC and CA line voltages- ±115.68 V- while the AC line voltage remains the same at 200.36 V. Interestingly, this is the only test wherein the phase is significantly altered, as can be seen by the BC and CA line voltages having a tighter angle between one another.

Part Three: Introducing Faults- Open Circuit

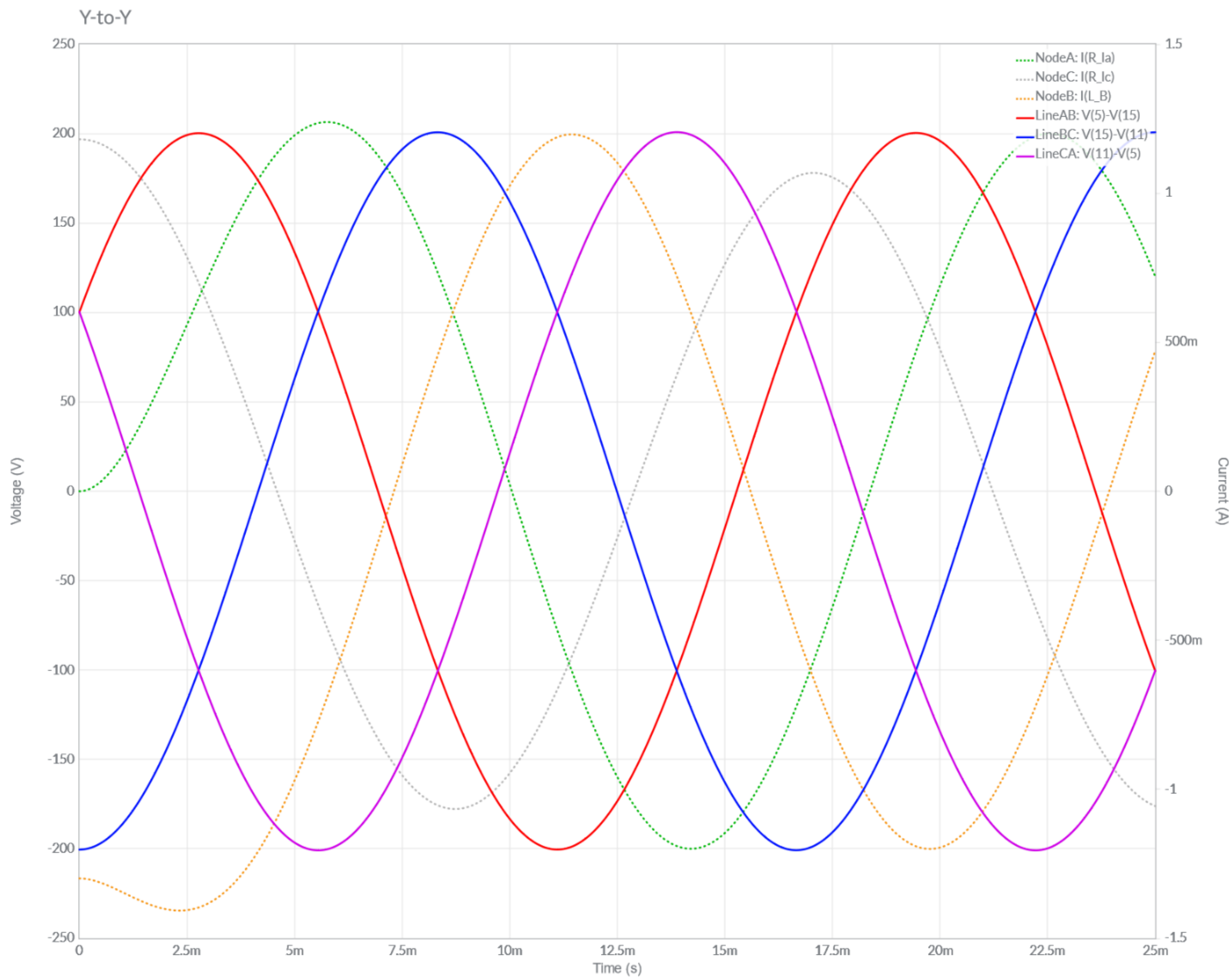
In this simulation, I opted to increase phase C’s load by 10 kΩ as a crude way to simulate an open circuit. The results are as follow:



Here, the differences between the line voltages are less pronounced, BC and CA only changing by ~0.85 V compared to default, and AB remains the same again. While the A and B line currents remain at ±1.2 A, the C line current comes down to about ±12 mA.

Part Four: Introducing Faults- Uneven Balance

In this simulation, I opted to- again- alter phase C's load, by increasing the resistor value to 85 Ω and the inductor value to 0.18 H. The results are as follow:



The variation from the default circuit is even less pronounced than Part Four: The line voltages of BC and CA only vary by ~0.5 V this time, and the C line current has new peaks at ±1.068 A.

Part Five: The Circuit in Y-to-Δ Configuration

In converting the configuration of the circuit from a Y-to-Y to a Y-to-Δ configuration, we must change the values of the load impedances. Thankfully, this is a simple ordeal, with the equation being:

$$Z_Y = \frac{Z_\Delta}{3} \quad (8)$$

Or rather, for our purposes for finding a Z_Δ value:

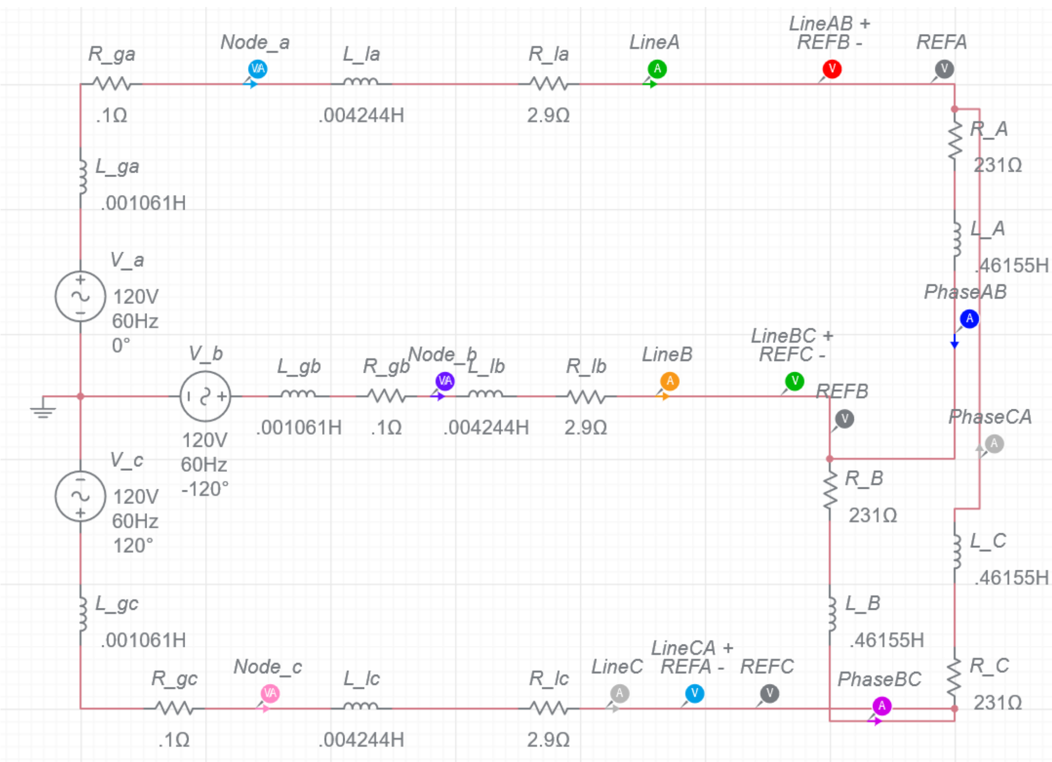
$$Z_\Delta = 3Z_Y \ (9)$$

And of course, this means we must multiply our current load values, both resistor and inductor, by 3. Thus, our values are:

$$R_{load} = 231\ \Omega$$

$$L_{load} = 0.46155\ H$$

Finally, our circuit will look like this:



Once again, the first step in finding the voltage across each impedance is to find the values of those impedances in Ohms. Thankfully, since the calculations are done one branch at a time, as in with the Y-to-Y values in place, we can re-use the same impedance values as before:

$$Z_{TA} = 100\Omega$$

$$\phi_A = 36.87°$$

From here, finding the A-phase's line current is just as simple as last time, reusing equation 7:

$$\boldsymbol{I}_{aA} = \frac{120\angle 0°}{100\angle 36.87°} = 1.2\angle - 36.87°\ A$$

Phase B and C are the same, just with a phase-shift of ±120°:

$$\boldsymbol{I}_{bB} = 1.2\angle - 156.87°\ A$$

$$\boldsymbol{I}_{cC} = 1.2\angle 83.13°\ A$$

This time, the configuration allows the line and phase voltages to be the same, as opposed to the currents. Thus, to find the line voltage, we must first find it in the context of the Y-to-Y A-phase isolation:

$$\boldsymbol{V}_A = 1.2\angle - 36.87° * 96.4\angle 36.99° = 115.68\angle 0.24°\ V$$

Now we follow this with equation 7 to find the final value:

$$\boldsymbol{V}_{AB} = 115.68\angle 0.24° * \sqrt{3}\angle 30° = 200.36\angle 30.24°$$

At this point one can notice that the above line voltage is the same as the one in Part One of this report. So, for brevity's sake, I will leave it at that. Lastly, I will note that the voltage drops across the source and line impedances remain the same as well.

However, what is not the same is that there is now a phase current. The phase current can be calculated very easily with the following formula:

$$\boldsymbol{I}_{phase} = \left(\frac{1}{\sqrt{3}}\angle 30°\right)\boldsymbol{I}_{line}\ (10)$$

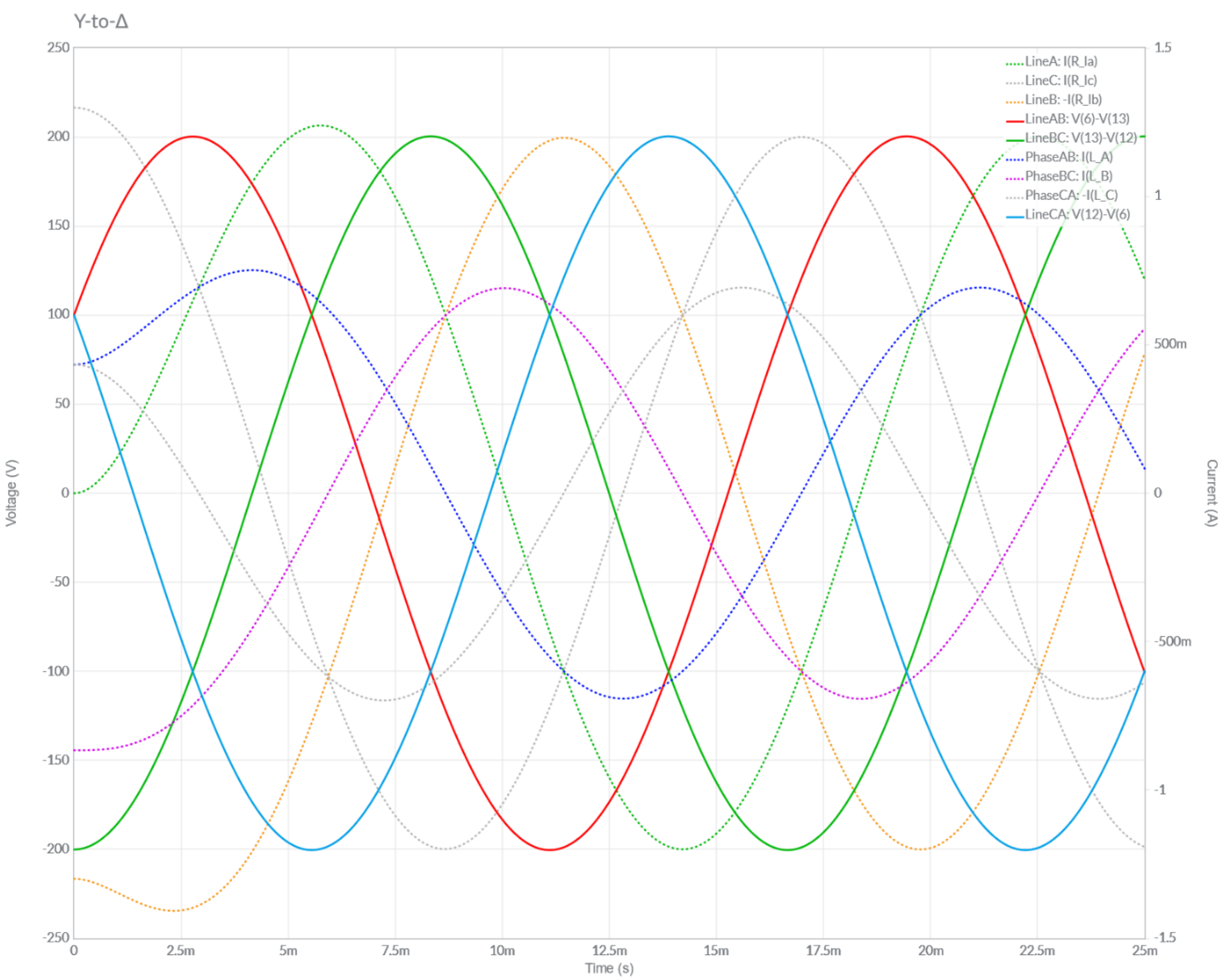
Therefore:

$$\boldsymbol{I}_{AB} = .6929\angle -6.87°\ A$$

$$\boldsymbol{I}_{BC} = .6929\angle -126.87°\ A$$

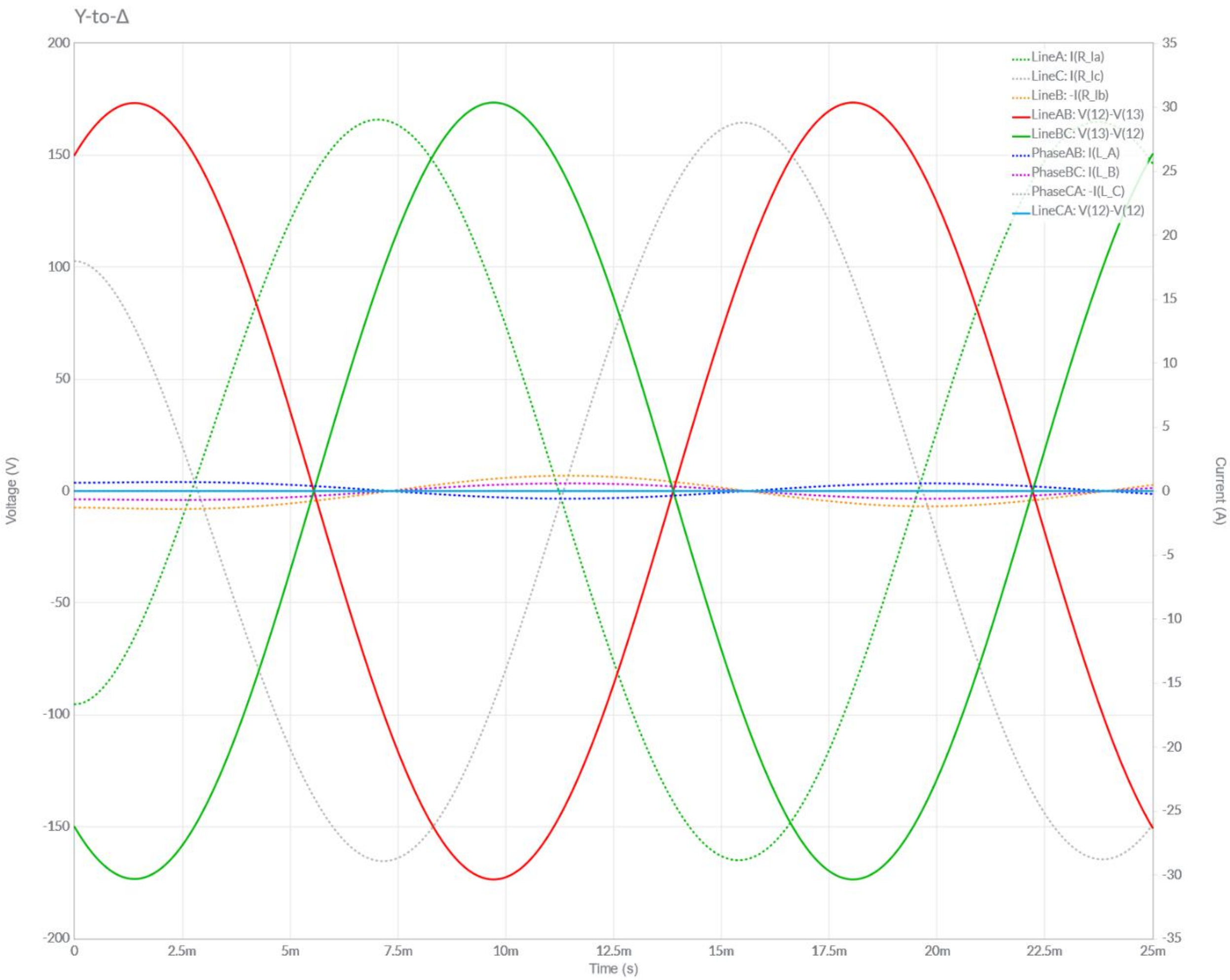
$$\boldsymbol{I}_{AB} = .6929\angle 113.13°\ A$$

Thusly, all the signals will seem as such:



Part Six: Introducing Faults- Short Circuit

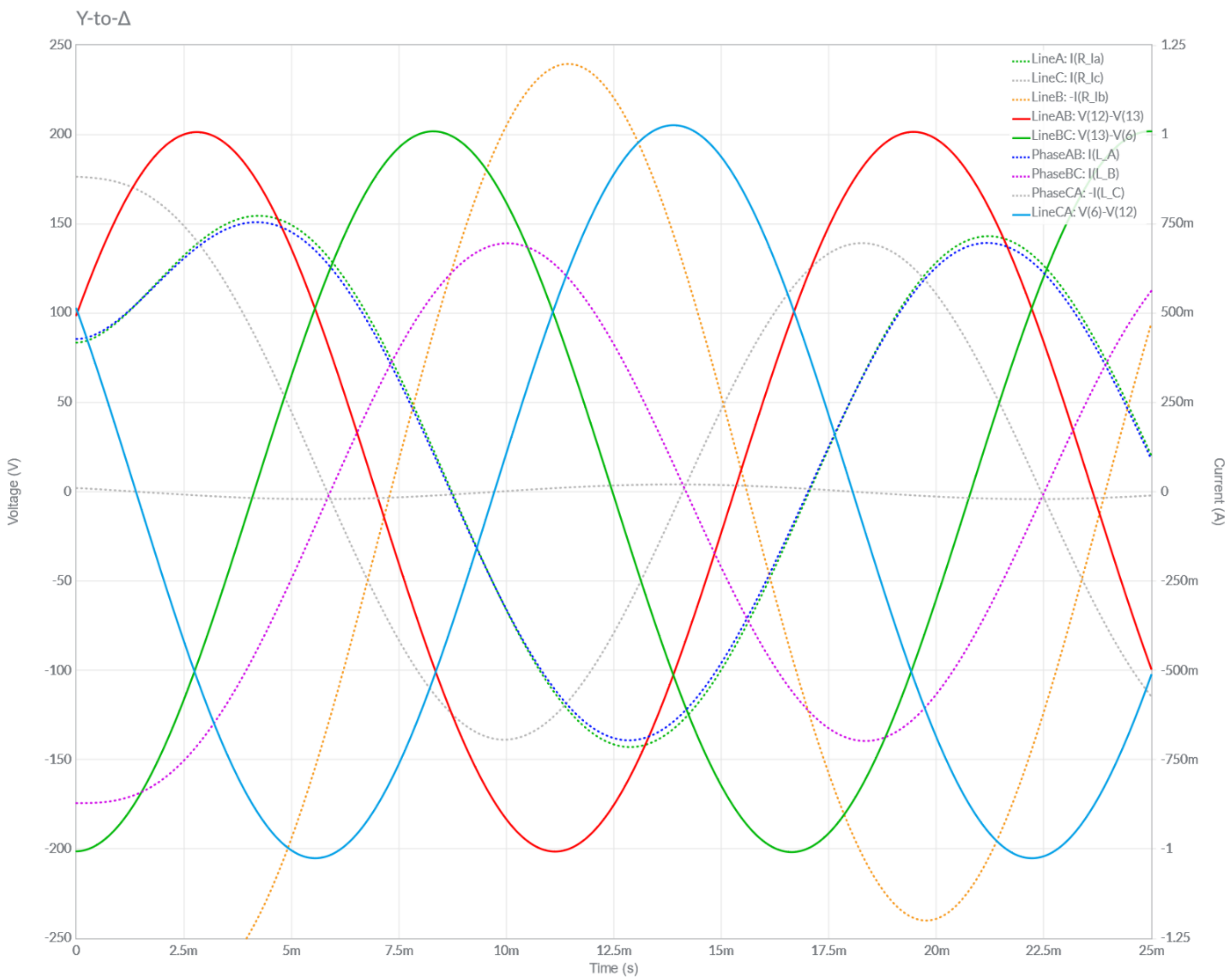
In this simulation, I opted to short phase C's load. The results are as follow:



In this case, a short circuit causes far more turmoil: the AB and BC line voltages drop their peak values to ±173.52 V and CA is a flat 0 V. Due to the absence of the CA line, the phase difference of the AB and BC lines is 180°. The B line current remains the same default ±1.2 A, but A and C line currents soar up to nearly ±29 A. Due to this, the A and C line currents nearly have a 180° phase difference between each other. The AB and BC phase currents lower slightly to ±0.6 A and the CA phase current reaches nearly- but not quite- 0 A.

Part Seven: Introducing Faults- Open Circuit

In this simulation, I opted to increase phase C's load by 10 kΩ as a crude way to simulate an open circuit. The results are as follow:



In this case, the voltages all throughout had minimal change, with the CA line having the largest difference from the default at ±205.17 V. The current, however, is all over the place, and to put it succinctly here is difficult.

Part Eight: Final Summary and Conclusions

For the summary, I would like to present the peak values in numerical form of the above graphs followed by my observations and personal remarks. Note that the values are not necessarily exact due to the imprecise nature of the simulation software.

Y-to-Y summary:

same/close *slightly different* *very different*	AB Line Voltage	A Line Current	BC Line Voltage	B Line Current	CA Line Voltage	C Line Current
Max/Min Default	200.36 V	1.2 A	200.36 V	1.2 A	200.36 V	1.2 A
Max/Min SC	200.36 V	1.2 A	115.68 V	1.2 A	115.68 V	33.28 A
Max/Min OC	200.34 V	1.2 A	201.2 V	1.2 A	200.74 V	0.012 A
Max/Min Unbal.	200.35 V	1.2 A	200.75 V	1.2 A	200.82 V	1.068 A

Y-to-Δ summary:

same/close *slightly diff.* *very diff.*	AB Line Voltage	A Line Current	AB Phase Current	BC Line Voltage	B Line Current	BC Phase Current	CA Line Voltage	C Line Current	CA Phase Current
Max/Min Default	200.36 V	1.2 A	0.693 A	200.36 V	1.2 A	0.693 A	200.36 V	1.2 A	0.693 A
Max/Min SC	173.52 V	28.863 A	0.6 A	173.52 V	1.2 A	0.6 A	0 V	28.796 A	~ 0 A
Max/Min OC	201.42 V	0.715 A	0.696 A	201.73 V	1.197 A	0.698 A	205.17 V	0.696 A	0.020 A

In general, it seems that with such systems, the largest issue that can arise is with the case of a short circuit, though the definition of an "open circuit" in the experiment being 10 kΩ may have played a part in the tame results. However, it does seem that through it all, a Y-to-Y system is far more stable when experiencing faults, perhaps due to the neutral line present at the intersection of the loads.

A final small note may be made that the summary tables (and, with a couple exceptions, my observations throughout this paper) do not include phase angles. This is due to the challenge that it is to gather that data with the tools at hand. However, it must be stated that each fault causes a different mixture of phase angles to arise, imperceptible (or not) as they may be to the eye. In practice, these micro or macro differences in phase angles between the variables are very important and may create unstable and unwanted behavior.