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Project Two: Filters

Introduction

In this report, I will detail how to design a 1st order active low-pass filter who's cutoff frequency is 10k rad/s, what happens when one cascades three of those filters, how to correct the cutoff frequency in that cascade design, and finally display a 3rd order Butterworth design so as to show the benefits of such a design over three simple active low-pass filters. Throughout the report, there will be Bode plots created via Matlab, circuit designs via Multisim, and all the necessary mathematics to prove the results found.

A note to make ahead of time, I used the real component values instead of ideal, so the graphs are a bit messy.

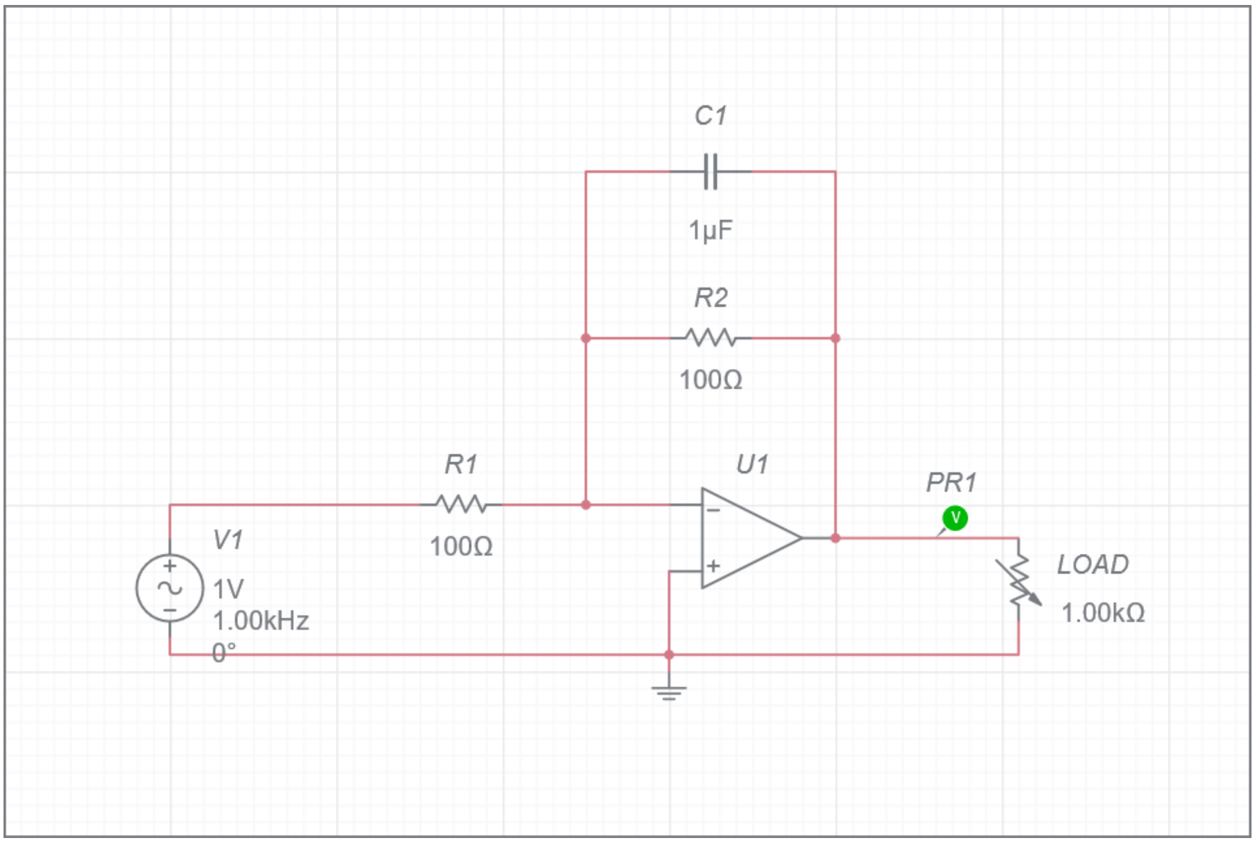
Part One: 1st Order Active Low-Pass

Designing a 1st order active low-pass filter is rather simple. Since we are creating a prototype filter, we do not want the circuit to cause any gain. Therefore, we will mirror R_1 and R_2's values. Then, to find the values for C and R_2 that create a 10k rad/s cutoff frequency, we simply use this formula:

$$\omega_c = \frac{1}{R_2 C} \quad (1)$$

So, since ω_c will be 10,000, then the R_2 multiplied with C must equate to 0.0001. Using rfcafe.com's[1] [2] tables of common values for resistors and capacitors, we come to the conclusion that a 100Ω resistor and a 1µF capacitor are decently common and give us our necessary value.

The circuit will thusly look like this:



The transfer function of this circuit will be as follows:

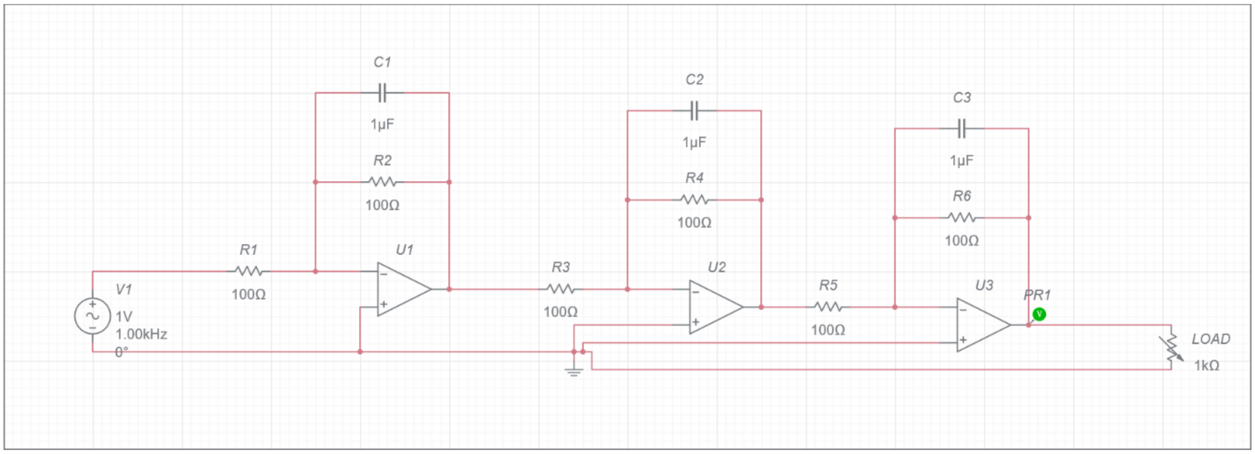
$$H(s) = \frac{-10000}{s + 10000}$$

However, it is known that for the Bode plot, we would like the equation in Bode plot form:

$$H(j\omega) = \frac{-10000}{j\omega + 10000}$$

Part Two: Cascade of Three Filters

Cascading the above circuit thrice looks as such:

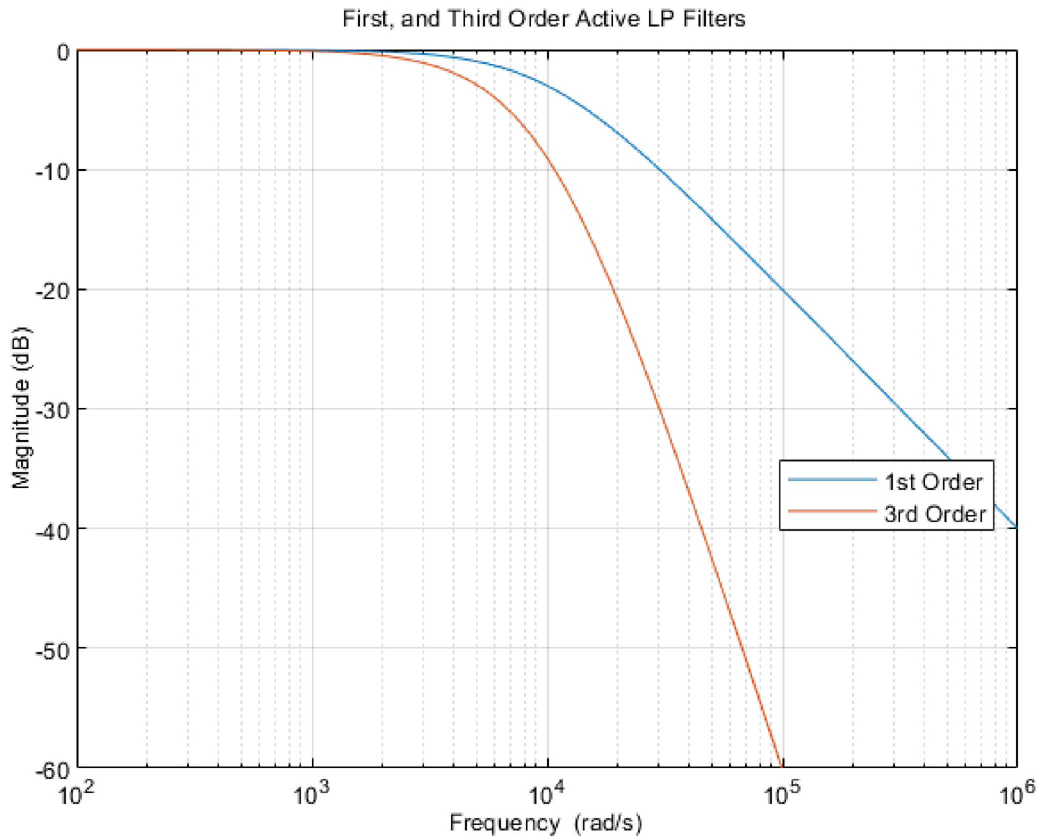


For the transfer function of the total circuit, we simple multiply the 1st order transfer function thrice:

$$H_3(s) = \frac{-10000^3}{s^3 + 30000s^2 + 300000000 + 10000^3}$$

$$H_3(j\omega) = \frac{-10000^3}{(j\omega)^3 + 30000(j\omega)^2 + 300000000j\omega + 10000^3}$$

And here we can see the Bode plot of the 1st order circuit and the 3rd order circuit:



As can be clearly seen, the cascaded circuit filters earlier and with a less steep slope all throughout, making the filter far less precise.

Part Three: Correcting the Cascade

To alleviate the error in the above design, we can attempt to alter the values of the capacitors and resistors. Our first step is to use the following formula to find the cutoff frequency offset:

$$\omega_{c3} = \sqrt{\sqrt[4]{2} - 1}$$

Which nets us:

$$\omega_{c3} = .5098 \text{ rad/s}$$

Therefore, to adjust for the cascade, we write:

$$\frac{10000}{.5098} = 19615.2$$

Thus, an individual filter's transfer function will look like:

$$H(j\omega) = \frac{-19615.2}{j\omega + 19615.2}$$

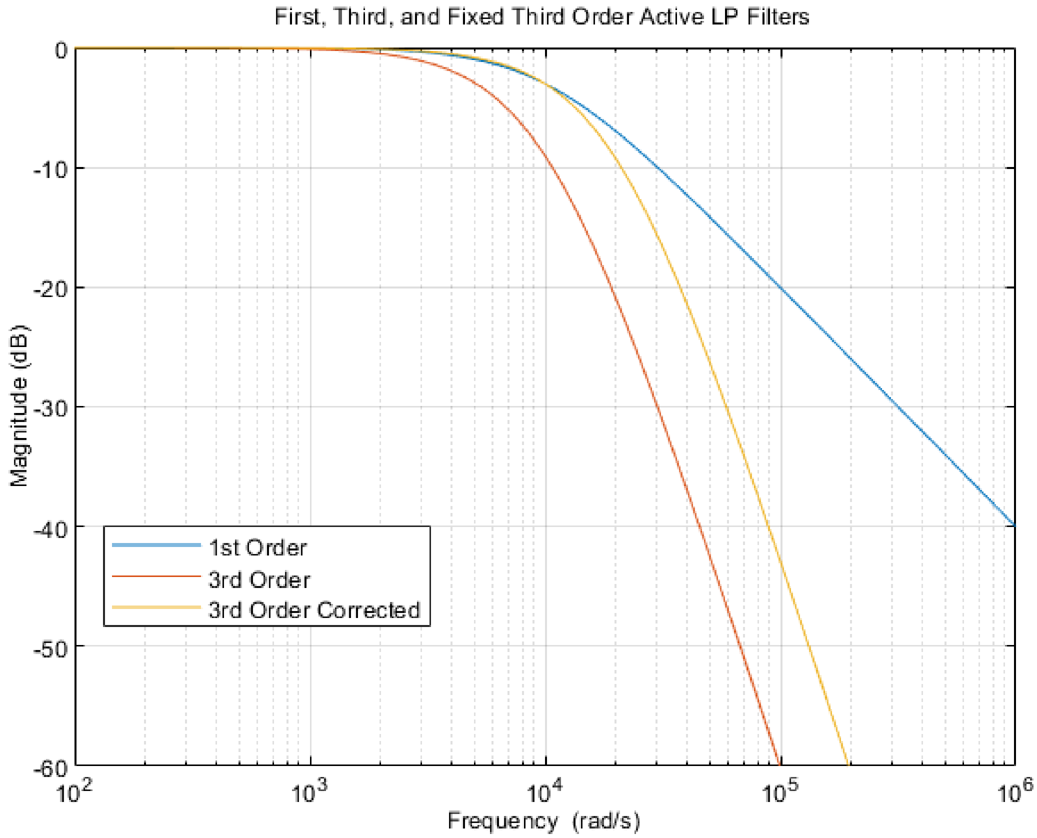
And multiplying three of them together:

$$H_3(j\omega) = \frac{-19615.2^3}{(j\omega)^3 + 58845(j\omega)^2 + 1.1542E + 09j\omega + 7.5471E + 12}$$

Now, to find the resistor and capacitor values, we simply reuse equation 1:

$$\omega_{c3} = \frac{1}{R_2 C_1} \Rightarrow R_2 = R_4 = R_6 = 51.1\Omega \;\&\; C_1 = C_2 = C_3 = 1\mu F$$

And of course, the plot of the 1st order, the cascade 3rd order, and the corrected cascade 3rd order filters:



And already the improvement is plainly visible, with the 1st Order and corrected 3rd Order graphs overlapping at the cutoff point of 10k rad/s, while also having a nicer, steeper slope.

Part Four: 3rd Order Butterworth Filter

To begin designing, we should first know the transfer function of a Butterworth filter on its own. Since we are designing a 3rd order filter, we will need one Butterworth low-pass filter circuit cascaded with a simple active low-pass filter. Thus, we will need the transfer function of just a single Butterworth and then we will multiply it with the simple filter.

A low-pass Butterworth filter on its own has the transfer function:

$$H_B(s) = \frac{\frac{1}{R^2C_1C_2}}{s^2 + \frac{2s}{RC_1} + \frac{1}{R^2C_1C_2}}$$

When prototyping a 3rd order Butterworth, we want $\frac{2}{RC_1} = 1$ and $\frac{1}{R^2C_1C_2} = 1^2$. However, in our case we want the values to equal 10,000 and $10{,}000^2$ respectively.

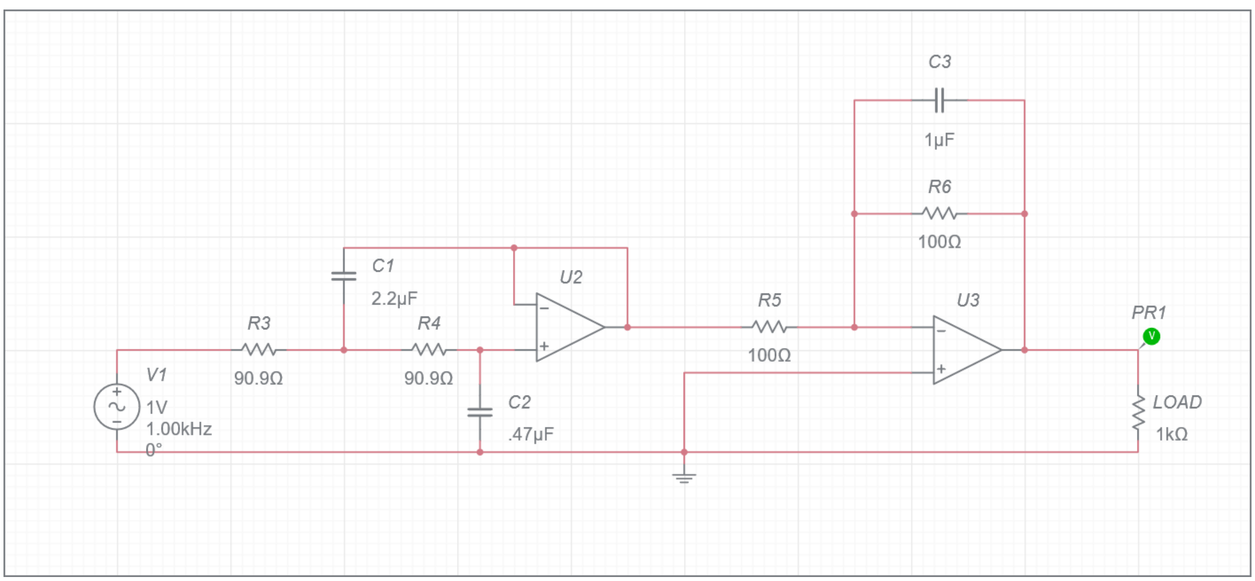
$$10000 = \frac{2}{RC_1} \Rightarrow R = 100\Omega \;\&\; C_1 = 2\mu F$$

Setting those values adjusts C_2 as well:

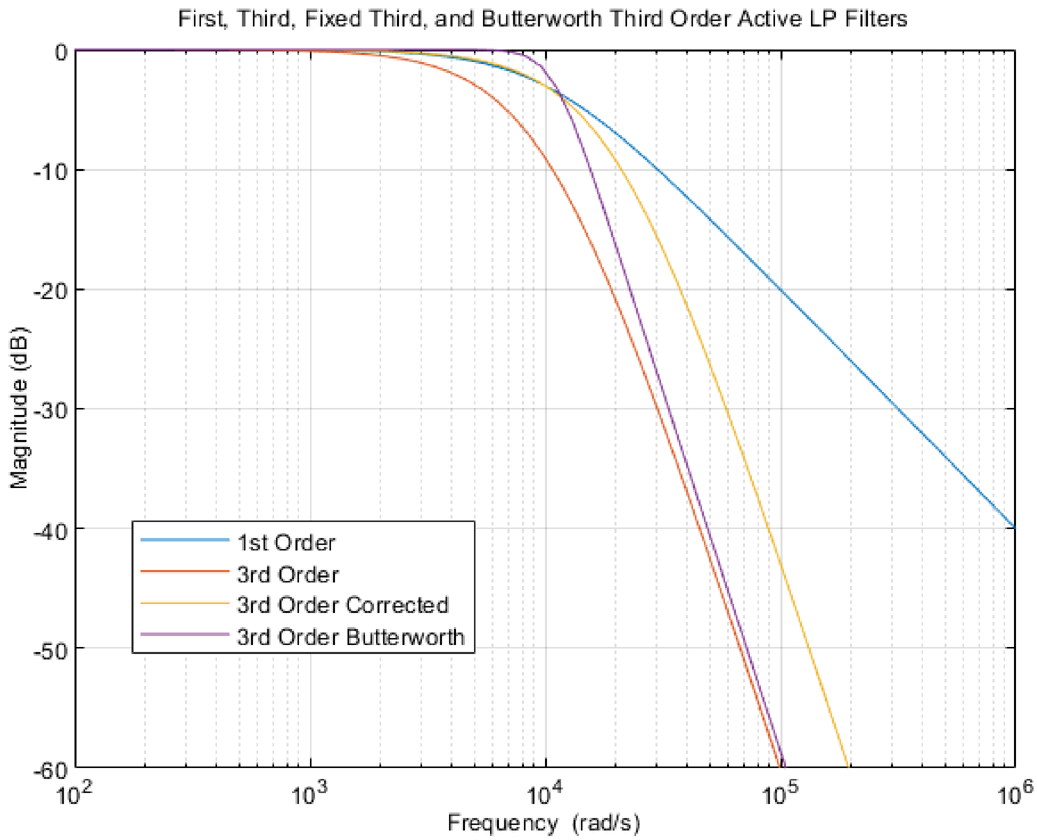
$$10000^2 = \frac{1}{R^2C_1C_2} \Rightarrow C_2 = .5\mu F$$

However, due to what parts are actually available, we must make concessions and set C_1 to 2.2μF, C_2 to .47μF, and R to 90.9Ω.

Thus our circuit looks like this:



And the Bode plots of this and the prior circuits is:



Of course, completely according to plan, the Butterworth filter performs the best of all, as it has the slimmest transition period. Unfortunately, due to the parts restriction, the filter's cutoff point is at about 11k rad/s, however an ideal Butterworth implementation hits the cutoff frequency exactly on the spot along with having the excellent curve.

Conclusion

There is no doubt that the more complex a system becomes, the better its outputs are. The simple 1st order filter did an alright job, and a (corrected) 3rd order of cascading design did better, but there is no doubt a Butterworth is as best we can touch right now. Assuming ideal components, all designs manage to have the cut-off filter at the right spot, but the major difference between each design is how clean the transitions, how sharp the curves, are. The ideal is a straight vertical line at the desired cutoff, but this is no ideal world.

With that said, here are some simple tables of the first five dBs of drop for each filter design:

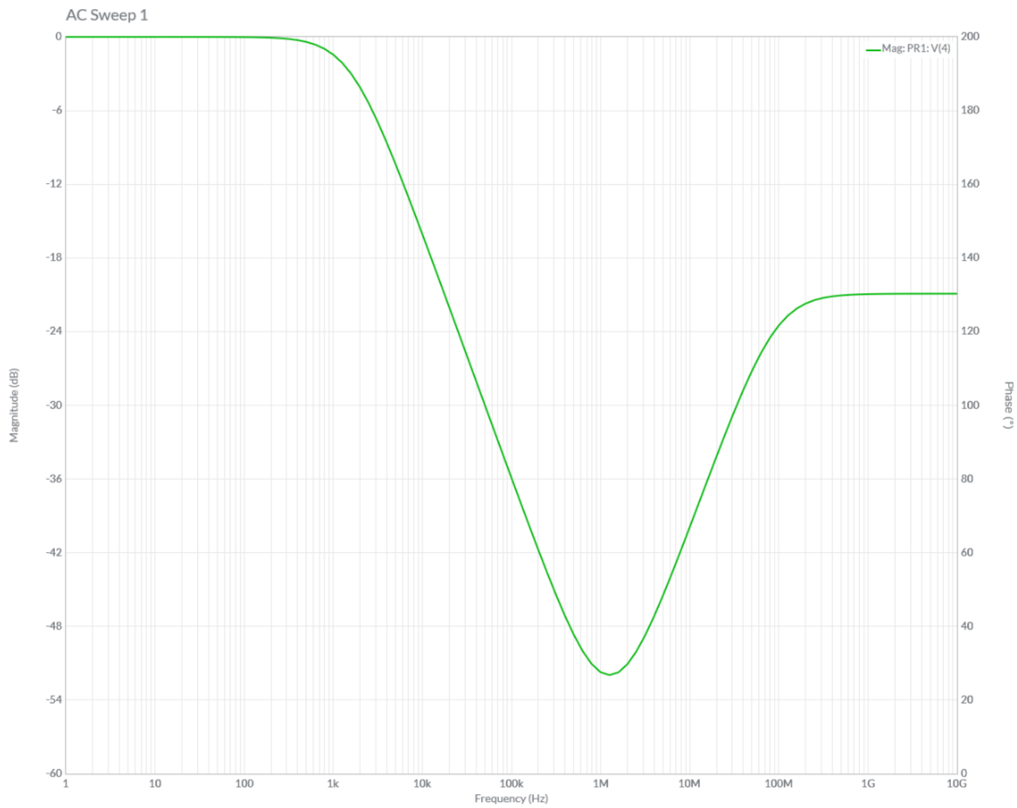
	1^{st} Order	3^{rd} Order	3^{rd} Order Butterworth Ideal	3^{rd} Order Butterworth Real
0dB	0	0	0	0
-1dB	5,110	5,550	7,880	8,980
-2dB	7,640	7,960	9,060	10,100
-3dB	9,970	9,940	9,999	11,000
-4dB	12,300	11,700	10,600	11,700
-5dB	14,700	13,400	11,300	12,400

The numbers corroborate the above graphs- the more advanced filters have a tighter angle. As can be seen, the tighter angle even allows the slightly off-center real-value Butterworth filter to have tighter allowance than the 1^{st} order or the cascade 3^{rd} order filters.

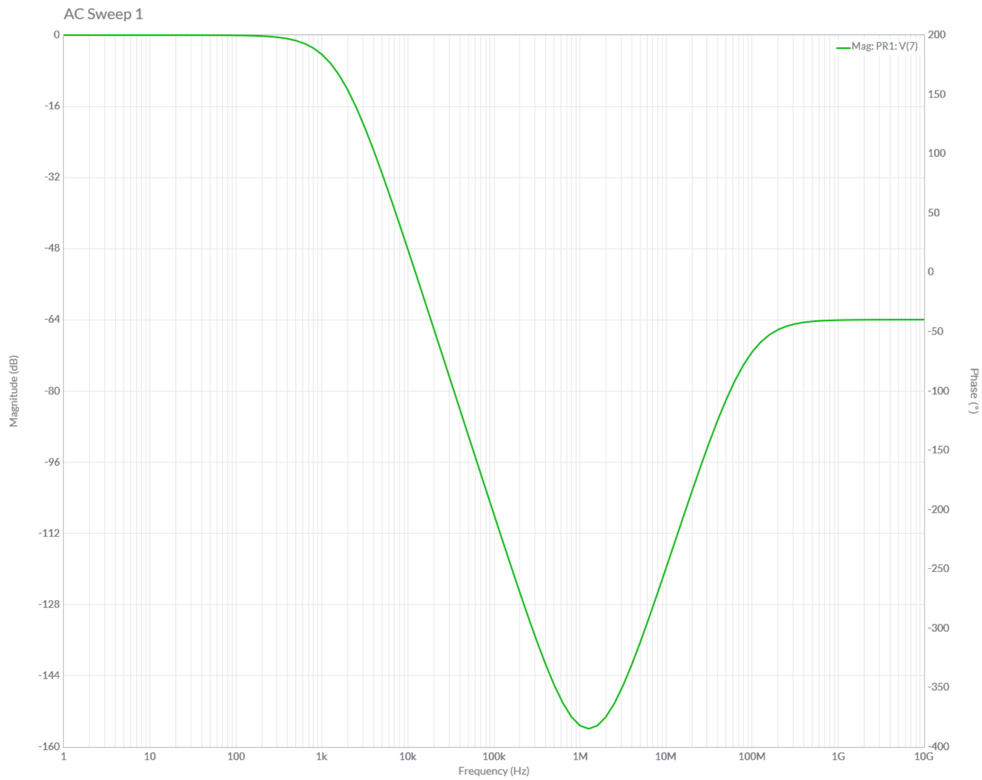
Part Five: Multisim Results

The only thing of note about these graphs is that they extend to a farther range and that the range is in Hz instead of rad/s.

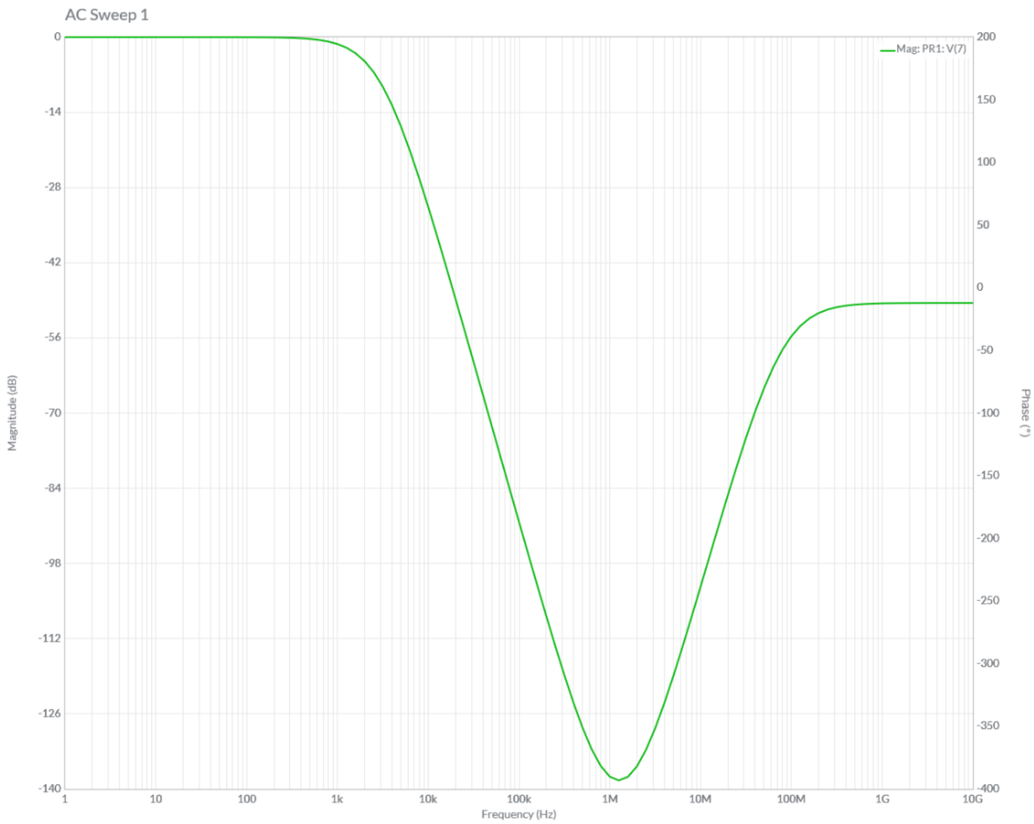
1st order:



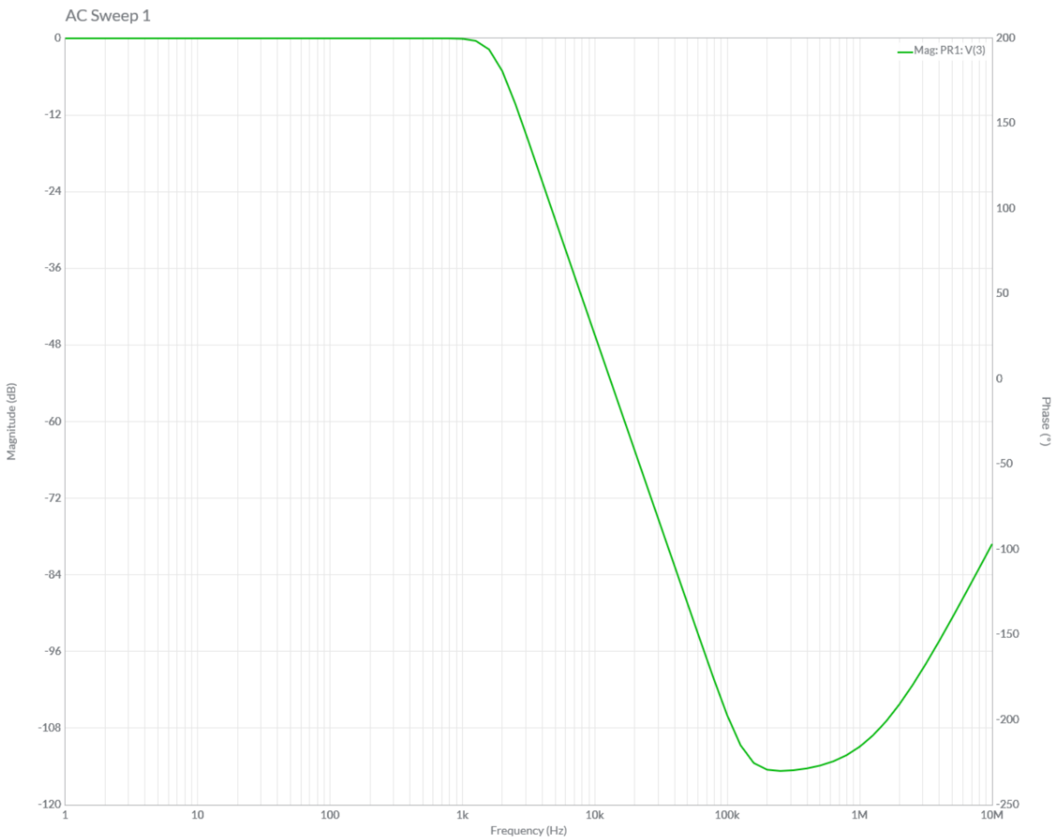
3rd order uncorrected:



3rd order corrected:



Butterworth:



AC Sweep 1
Mag: PR1: V(3)
Magnitude (dB)
Phase (°)
Frequency (Hz)
0
-12
-24
-36
-48
-60
-72
-84
-96
-108
-120
200
150
100
50
0
-50
-100
-150
-200
-250
1
10
100
1k
10k
100k
1M
10M